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A Descriptive Study on Mathematical Model of Shark’s Capabilities as a Successful Hunter

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A Descriptive Study on Mathematical Model of Shark’s Capabilities as a Successful Hunter

Nasser Yousefi¹, Mohammad Sadegh Aramli¹*
¹Young Researchers and Elite Club, Ardabil Branch, Islamic Azad University, Ardabil, Iran.

*Correspondence: msaramli@gmail.com

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Abstract
In this paper, we have studied the abilities of shark in the sea environment. Sharks are mysterious and misunderstood creatures that have fascinated the people for generations. This trunk will introduce students to the biology and behaviors of sharks. The main purpose of this paper is to discover this animal’s abilities, especially its olfactory bulbs, and the reasons which makes the shark a successful hunter. After studying the characteristics of this animal, we will analyze each of them to answer: “how we can use these capabilities in our life?” Accordingly, we will focus on shark’s swimming and olfactory bulbs to achieve mathematical model. This model can be used in several engineering and other problems in human life.

Keywords: Shark; Olfactory bulbs; Odor particle; Swimming.

1. INTRODUCTION

There are more than 350 species of sharks in the world. Sharks belong to a big group of fishes named Chondrichthyes, which in Greek means “cartilage fish.” This group also includes skates and rays, all of which have skeletons made of cartilage, not bone. The word “shark” is probably derived from the German word “schurke,” which can be translated as “villain” or “greedy parasite.” Sharks are believed to have evolved in the Devonian Period, over 400 million years ago. They swam the seas and oceans 200 million years before the dinosaurs, where they have remained relatively unchanged for the past 65 million years. But, what is the secret of this animal which helps them to survive over million years?

This animal has a skeleton made of cartilage, the same material that our ears and noses are made of. Cartilage is less dense than bone and helps the shark to be more buoyant and flexible. Sharks have no ribcage; they are held each other by skin and muscle. This can lead the shark to be more susceptible to internal damage [1].

The brain of sharks is larger than most cold-blooded animals. With this larger brain comes a vast amount of sensory information. In addition to the five senses used by humans, sharks possess a unique sensory adaptation. This animal utilizes all of its senses to locate food.

The eyes of sharks are small, but they can see rather well, even in dim light. Color vision is believed to be somewhat limited. Sharks’ eyes are very sensitive to light. They are designed for seeing in the dim light underwater. It means that this ability is not so strong which can be distinguished. Also, same as the humans, this animal have upper and lower eyelids. In addition, they also possess third eyelids that cover the entire eye. Deepwater sharks generally have bigger eyes than shallow water sharks [2-4].

Sometimes, sounds help the sharks to locate the food. Actually, this ability of sharks is strong, where some can hear prey in the water from 3,000 feet away. Their internal ear can detect sound as well as vibrations in the water, such as the thrashings of sick fish.

Swimming is the important ability of shark which helps them to move to the food’s location. Shark’s bodies are fusiform (streamlined and torpedo shaped). They have five different kinds of fins that they use to lift, stabilize, and propel themselves. The caudal fin, or tail fin, can be used for turning and for propulsion. Unlike most of the fishes, the shark's backbone extends well into the tail, making it very strong. The erect dorsal fin on a shark’s back is used for balance. The dorsal fin often is seen above the water surface when a shark is swimming. The second dorsal fin controls rolling. The front fins, or pectoral fins, are much stiffer than in other fish. The shark can change the angle of these fins to swim either up or down; they cannot swim backwards. Stability is provided by the pelvic fins. Some sharks have an anal fin to provide extra stability [5-7].

Many sharks are rather sluggish swimmers, although some can swim very fast. One kind of sharks can swim from 20 to 30 miles per hour. Unlike most bony fish, sharks have no swim bladders to keep them afloat. They can use oil in their liver, which can be more than 15% of their total body weight, for buoyancy. To avoid sinking, sharks must constantly swim in a slightly upward direction.

Additionally, sharks have an extremely acute sense of smell. Nearly two-thirds of a shark’s brain is devoted to the sense of smell. They can detect minute quantities of certain substances, especially blood, in the water. Fish give off a certain odor when they are in distress, which is easily detected by sharks. They can detect odors up to one mile away.

In this paper, engineering problem is not the main issue; we will focus on swimming and smell abilities to find a mathematical model. So in the following we will introduce the mentioned capabilities in literature as; Section 2, introduce the shark’s body. Swimming model of shark is presented in Section 3. Section 4, presents the smell abilities of shark by proposed model. Finally, Section 5 conclude the paper.
2. COMPONENTS OF SHARK BODY

The class Chondrichthyes consists of cartilaginous fishes (sharks, ratfish or chimaeras, and skates and rays) that are widely distributed in the world's oceans. Some species enter freshwater, and a few, like the rays Paratrygon motoro and Himantura signifer, live exclusively in freshwater. Unlike bony fish, shark's body is complex made up of flexible collagenous fibers and arranged as a helical network surrounding their body. Where, it works as an outer skeleton, providing attachment for their swimming muscles and thus saving energy. The shark's dermal teeth provide them hydrodynamic advantages as they help in bringing down turbulence when swimming. Figure 1 illustrates the terminology used to identify morphological features of shark [8-10].

Most of the times vortices and whirlpools are formed behind the placoid scales as a shark swims which can help the shark swim efficiently. The fact that sharks are completely covered in tooth-like structures may seem alarming, but not only are they beneficial when it comes to swimming, but they also form a barrier of protection. Interestingly enough, however, while the dermal denticles are arranged in a pattern on the shark, they do not grow as the shark grows. Instead, the shark just sprouts more placoid scales as necessary. Hence, age estimation cannot be determined by the scales of the shark, although, the ages of other fish can be calculated this way [11].

Like other fish, the anatomy of the sharks includes gills to aid in respiration. Located on the side of a shark's head are five to seven gill slits; in order for the gas exchange to occur efficiently, water has to consistently flow over the gill slits. Once the shark allows water to enter their mouth, it goes through the pharynx, over the gills, and finally leaves through the actual gill slits. In some types of this animal, spiracles are present as first gill slits. These slits are located behind the eyes, and they are used to send blood through a separate, unique blood vessel immediately to the eyes and brain of the shark. Although this feature of a shark's anatomy sounds very useful, they are not present on many sharks. Spiracles are mainly found on sharks that typically dwell near the seabed, otherwise known as sedentary sharks. The faster swimming sharks usually do not possess this feature, and if they do happen to have spiracles, they are most likely small in size.

3. SWIMMING MODEL OF SHARK

As mentioned, unlike most fishes, sharks do not have swim bladders to help them stay afloat. They must constantly swim in a slightly upward direction to keep from sinking. They use strong tail fin for propulsion. Therefore, the shark is displaced to the best Odor Particle (OP). Where, according to the initial OP and the information of shark position, the best answers will be generated. In this paper, we classified some abilities of shark as operator for using them in mathematical models [12-16].

3.1. Swimming Model

The main properties of water as locomotion medium that have played an important role in the evolution of fish are its incompressibility and its high density. Since water is an incompressible fluid, any movement executed by an aquatic animal will set the water surrounding it in motion and vice versa. Its density (about 800 times that of air) is sufficiently close to that of the body of marine animals to nearly counterbalance the force of gravity. This has allowed the development of a great variety of swimming propulsions, as weight support is not of primary importance [17]. The fin dimensions normal and parallel to the water flow are called span and chord, respectively. If we consider the movement of shark by the following equation:

\[ V_i = \eta_k R_i \nabla (OP)_i \delta_{i} \quad i = 1, ..., NP, \quad k = 1, ..., k_{\text{max}} \]  

(1)

Swimming involves the transfer of momentum from the fish to the surrounding water (and vice versa). The main momentum transfer mechanisms are via drag, lift, and acceleration reaction forces. Swimming drag consists of the following components:

1) Skin friction between the fish and the boundary layer of water (viscous or friction drag): Friction drag arises as a result of the viscosity of water in areas of flow with large velocity gradients. Friction drag depends on the wetted area and swimming speed of the fish, as well as the nature of the boundary layer flow.
2) Pressures formed in pushing water aside for the fish to pass (form drag). Form drag is caused by the distortion of flow around solid bodies and depends on their shape. Most of the fast-cruising fishes have well streamlined bodies to significantly reduce form drag.

3) Energy lost in the vortices formed by the caudal and pectoral fins as they generate lift or thrust (vortex or induced drag): Induced drag depends largely on the shape of these fins.

The main factors determining the relative contributions of the momentum-transfer mechanisms to thrust and resistance are: 1) Reynolds number; 2) reduced frequency; and 3) shape [18].

The Reynolds number (Re) is the ratio of inertial over viscous forces, defined as:

$$Re = \frac{LU}{\nu}$$  \hspace{1cm} (2)

Where, \(L\) is a characteristic length (of either the fish body), \(U\) is the swimming velocity, and \(\nu\) is the kinematic viscosity of water.

Where, \(Re\), acceleration reaction, pressure drag, and lift mechanisms can all generate effective forces is presented in Figure 2.

The reduced frequency \(\sigma\) indicates the importance of unsteady (time-dependent) effects in the flow and is defined as:

$$\sigma = \frac{2\pi f}{U}$$  \hspace{1cm} (3)

Where \(f\) is the oscillation frequency, \(L\) is the characteristic length, and \(U\) is the swimming velocity. The reduced frequency essentially compares the time taken for a particle of water to traverse the length of an object with the time taken to complete one movement cycle.

Finally, the shape of the swimming fish and the specific propulsion utilized largely affect the magnitude of the force components. The relationship is well documented for steady-state lift and drag forces, but relatively little work has been done on the connection between shape and acceleration reaction.

A common measure of swimming efficiency is Froude efficiency, defined as:

$$\eta_f = \frac{\langle T \rangle U}{\langle P \rangle}$$  \hspace{1cm} (4)

Where, \(U\) is the mean forward velocity of the fish, \(\langle T \rangle\) is the time-averaged thrust produced, and \(\langle P \rangle\) is the time-averaged power required.

Mechanical thrust power for a fish swimming at an average speed \(U\) is calculated [19] as:

$$P_t = m W_0 U - \frac{m \rho U^2}{2 \cos \theta}$$  \hspace{1cm} (5)

Where,

$$m = \left( \frac{B}{2} \right)^3 \pi \rho$$  \hspace{1cm} (6)

**Figure 2:** Diagram showing the relative contribution of the momentum-transfer mechanisms for swimming vertebrates, as a function of \(Re\).
Which $m$ is the added mass per unit length ($B$ is the trailing edge span and $\rho$ the density of the water), while:

$$W = \frac{f A \pi}{1.414}$$  \hspace{1cm} (7)

Which $W$ is the rms value of the lateral speed of the trailing edge ($f$ is the frequency of the caudal fin oscillations and $A$ is their amplitude). The velocity $\omega$ given to the water at the trailing edge is obtained as

$$\omega = W \left(1 - \frac{U}{V}\right)$$  \hspace{1cm} (8)

where $V$ is the velocity of the propulsive wave. Finally, $U$ is the angle of the trailing edge to the lateral plane of motion. Filmed sequences of the swimming fish are used to determine these parameters.

Most fish generate thrust by bending their bodies into a backward-moving propulsive wave that extends to its caudal fin, a type of swimming classified under body and/or caudal fin (BCF) locomotion. In the anguilliform mode, the shark’s body participates in large amplitude undulations, is presented in Figure 3.

Fish swimming in the thunniform mode is characterized by a stiff caudal fin, shaped like a tapered hydrofoil of a moderate sweepback angle with a curved leading edge and a sharp trailing edge which is presented in Figure 4a.

The caudal fin performs a combination of pitching and heaving motions, tracing an oscillating path as the fish moves forward, characterized by a peak-to-peak amplitude $A$, a tail-beat frequency $f$, and a wavelength $\lambda$ (Figure 4b). As the fin moves along this trail, its forward velocity $U$ is the same as that of the fish, while its lateral velocity $W$ changes in time. Other important parameters of its motion include the angle of attack $\alpha$ (with respect to its trail) and the feathering angle $\phi$ (Figure 4b). Feathering is the angle between the fin trail and the overall path of the fish. Both $\alpha$ and $\phi$ change as the caudal fin sweeps laterally in order to obtain maximal thrust during the whole of the fin-beat cycle [20-22].

For a thunniform swimmer, the reduced frequency $\sigma$ represents the ratio of the time to swim a distance equal to the caudal fin chord (usually calculated as $c = b/S$) to the tail beat period:

$$\sigma = 2\pi \frac{fc}{U}$$  \hspace{1cm} (9)

The proportional feathering parameter $\theta$, is defined as the ratio of slopes between $\alpha$ and $\phi$ and can be computed as:

$$\theta = \frac{\alpha_{\max} U}{W_{\max}}$$  \hspace{1cm} (10)

**Figure 3: Gradation of BCF swimming movements from thunniform mode (Shark mode).**
Where $\alpha_{\text{max}}$ is the angle of attack in radians (the slope of $\alpha$), $W_{\text{max}}$ the maximum lateral velocity of the fin, and $U$ is the swimming speed.

Since shark has inertia, its acceleration is limited. Thus, its velocity depends on the previous velocity.

### 3.2. Local Search

The swimming speed of sharks is low, whereas sometimes they turn around like a local search. But when they are chasing the food or escaping enemies, some can swim very fast, up to 45 miles per hour. Additionally, the shark’s swimming process in nature is like a local search. Furthermore, the water flows into the sea can deflect the odor particles to other places. In this situation, the shark doesn’t stop the searching process. The shark searches around the weak particles as a local search to find the better one. Accordingly, sharks search the sea environment as a local search to opt the best way to continue the search procedure. From optimization viewpoint, shark implements a local search in each stage to find better candidate solutions.

### 4. SMELL ABILITIES OF SHARK

The receptors for taste and smell are stimulated by certain types of chemicals. Two senses are much alike, and when we speak of a taste sensation we are often referring to a compound sensation produced by stimulation of both taste and smell receptors. The first one is; why hot foods often have more “taste” than cold foods is that they vaporize more: the vapors pass from the mouth upward into the nasal passages and stimulate smell receptors. And the second one; why a person with a cold cannot “taste” foods well is that, with nasal passages inflamed and coated with mucus, the smell receptors are essentially nonfunctional. Conversely, some vapors entering our nostrils pass across the smell receptors and enter the mouth, where they stimulate taste receptors [23, 24].

In sharks, it extracts oxygen from the water to metabolize their food. The gills are found on five to seven vertical arches that form the walls of the external gill slits. The olfactory organ of cartilaginous fishes includes a chamber inside a cartilaginous capsule. Incurrent and excurrent nostrils on the ventral snout allow water to flow over the olfactory mucosa. The anterior margin of the nasal flap is enough diverse that it was used in species identification until researchers learned that within-species variability is also high. In chimaeras, the two olfactory chambers are adjacent, with a cartilaginous septum which separates them down the midline. The excurrent nostrils are located medially above the mouth, and the excurrent nostrils open laterally near the edge of the mouth. In a separate chamber beneath the gills, the heart pumps oxygen-depleted blood to capillary beds in the arches. Water is pumped into the mouth and out across the gills counter to the direction of blood flow. This exchange system greatly enhances the rate and efficiency of oxygen diffusion into the blood. In sedentary animals, flow is driven by respiratory movements [24-26]. So, water is driven over the olfactory epithelium by swimming movements, as illustrated in Figure 5.

The number of odorant molecules that cross the olfactory epithelium is:

\[
N(\text{moles sec}^{-1}) = C(\text{moles sec}^{-3}) A_n(\text{cm}^2) V(\text{cm sec}^{-1})
\]

where $C$ is the concentration of odor molecules, $A_n$ is the area sampled by the naris, and $V$ is the velocity of odorant over the olfactory rosette (Figure 7). The stimulus strength ($S$) is:

\[
S = N(\text{moles sec}^{-1}) A_n(\text{cm}^2) t(\text{sec})
\]
where the number of receptors is assumed to be proportional to the area of the sensory epithelium ($A_r$), and $t$ is the time a molecule spends in the rosette.

The notion that animals respond to bilateral odor concentration differences is based on the fact that odor dilutes and diffuses gradually away from the source, and on the commonly held but erroneous idea that this causes a measurable concentration gradient. This idea does not take into account the chaotic nature of most odor dispersal processes. Odor plumes show chaotic intermittency, with the concentration variance several orders of magnitude greater than the concentration mean. Therefore, a spatial concentration gradient can be obtained only by averaging. However, it typically requires many minutes for the concentration averaging process to reach a stable mean. For most animals, this is much too slow to be useful for tracking prey or mates. When one naris received a $2\times$ stronger odor pulse, the animals turned toward the side receiving the stronger stimulus. But, it is likely that concentration and timing differences were confounded. The odors in their experiment were preloaded as a discrete bolus into the long tubing, by seawater both ahead of and behind the odor. This caused dilution of the leading and trailing edges of the odor bolus: it reached 50% of the applied concentration 7 seconds after initiation of odor delivery. Thus, the high concentration side would have reached response threshold before the low concentration side, and the animals received bilateral differences not only in the concentration but also—unintentionally—in the arrival time of detectable levels of odor at each naris [27, 28].

So, we can categorize three kinds of operators for odor particles in literature as; selection original odors, diffusion, and convection to create a new generation.

1) **Selection Original OP**: selection chooses the best odor particles in the population to create a new population for the next generation based on their fitness.

2) **Diffusion**: The diffusion of the odor particles in the water is done by two methods. The first way is molecular diffusion (crossover) and the second one is convective (mutation). By spreading the blood in the water, they mix with the water molecules. So the water molecules achieve weak smell ability. So the crossover operator should be considered in a new population. Also, instead of the single-point crossover, we adopt the two-point crossover. Figure 6, shows the molecular diffusion in the search environment. Also, the chemical potential gradient is calculated as:

$$ \nabla \mu = \frac{\partial}{\partial x} \mu_t + \frac{\partial}{\partial x} \mu_j + \frac{\partial}{\partial x} \mu_k $$ \hspace{1cm} (13)

3) **Convection**: As mentioned, the diffusion may be created by convection. So, the position can be changed by the water flow or other external forces. Figure 7, shows the convection of new blood for the sea environment. Since the mean molecular velocity in liquids and gasses are about equal at the same temperature, whereas the mean free path is of the order of one molecular diameter, it would be expected that the diffusion coefficients in liquids are only about 10 times less than that in gasses. However, the diffusion coefficients of small molecules in liquids are about four orders of magnitude lower than that in gasses. For example, the diffusion coefficient of nitrogen in water is $1.88 \times 10^{-9} \text{m}^2/\text{s}$. An estimate for the diffusion coefficient can be obtained using the Stokes-Einstein equation for the diffusivity [29]:

$$ D = \frac{kT}{3\pi \eta d} $$ \hspace{1cm} (14)
where,
\[ d \] is the diameter of the molecule that is diffusing
\[ \mu \] is the viscosity of the suspending fluid
\[ T \] is the absolute temperature
\[ k \] is the Boltzmann constant

Approximately, the distance \( d \) that molecules diffuse depends on the square root of the time, where;

\[ t = \frac{d^2}{D} \]  

where, \( D \) is the “diffusion constant.”

By changing this equation with algebra;

\[ \frac{d}{t} = \frac{D}{d}, \text{ but } \frac{d}{t} \text{ is the average speed } \nu, \text{ so} \]

\[ \nu = \frac{D}{d} \]  

If the distance has been increased, the velocity falls. The longer the distance you want the molecules to go, the slower they will go on average. A typical diffusion constant for a medium-sized molecule in water is about \( 5 \times 10^{-6} \). If the smell travels a quarter mile, or a little less than \( 5 \times 10^4 \), we get a speed of;

\[ \nu = \frac{5 \times 10^{-4} \text{cm}^2 / \text{s}}{5 \times 10^4 \text{cm}} \cdot 10^{-10} \text{cm/s} \]

This speed is fantastically slow and says that it would take 10 million years for blood to diffuse out to a shark a quarter mile away. Diffusion is very important for small things, like bacteria, since the speed increases as you get smaller.

Why does it take so incredibly long for a random motion to spread particles around through a large region? Consider two small patches of the ocean right next to each other. At first, there’s a patch with lots of blood right next to a patch with very little blood. As particles move randomly, lots of blood particles move into the bloodless region without many going back the other way. But after a while, the blood is spread out pretty evenly over an area of a few cubic meters. By this point, if you take a little patch of water, it has almost the same, low amount of blood as the patch next to it. Thus, random motion sends some blood particles to the left, some to the right, and they’re almost equal, so there’s very little motion of the blood overall. The further you want the blood to spread, the more evenly distributed it becomes, and so the spread advances slower and slower.

So diffusion is not responsible for the spread of odors on human or shark-sized scales.

Waves also do not carry the smell to a significant extent. Here’s a picture of how a wave travels along the surface of water.

The point is that each individual point just moves in a small circle as the wave passes. Just like fans can do “the wave” at a sports game without anyone actually leaving their seat, waves travel through water without taking any of the actual water (or smell molecules) with them [30].

In practice, smell spreads through water or air via mixing or currents. The water is always in motion: tides, temperature gradients, wind, fish swimming, Coriolis forces, and so on; all keep the water moving (though on different scales). This motion
of the water carries smell molecules with it. The more the motion, the faster the smell will travel, but there is no fixed speed, and in fact it will be different in different directions depending on which way the current is traveling. This is why hunters care about being upwind or downwind of prey, since wind will carry their smell.

5. CONCLUSION

In this research, we have tried to present some of the important abilities of shark which make it a successful hunter. Ability to find food is the most important factor behind shark’s survival for over million years. Finding food involves swimming and smelling abilities of sharks. The mentioned abilities of sharks can be applied for the betterment of human life. For this purpose, we need a mathematical model to apply them in algorithms or capturing them in engineering problems. There are several factors which have not been considered in this paper that can be studied in the future.

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Designed the study, collected and analyzed the data: NY; Prepared the manuscript: MSA.

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